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Unsteady Transonic Flow in Two-Dimensional Channels

T. C. ADAMSON JR.
M. S. LIOU

October 1978

Final Technical Report
Prepared for
Naval Air Systems Command
Washington, D.C. 20361
Contract N00019-77-C-0263

Department of Aerospace Engineering

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Unsteady transonic flow with a shock wave in a two-dimensional channel is considered. Solutions valid in a thin inner region enclosing the nozzle throat, where previous solutions showed singular behavior, are presented. A brief description of a computer movie is given; this movie shows solutions for unsteady transonic channel flow for two different cases in which the shock wave, as a result of pressure oscillations impressed downstream of the nozzle throat, moves to and upstream of the throat, disappears, and reforms at the throat. An analysis showing how results derived for the interaction → next page

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between a shock wave and a turbulent boundary layer in steady flow may be used in the corresponding quasi-steady flow problem, is presented; a discussion of how these results may be used to deduce the order of the distance from the shock wave to the separation point and the time characteristic of the life of a shock induced separation bubble in unsteady flow is given.

UNSTEADY TRANSONIC FLOW IN TWO-DIMENSIONAL CHANNELS

by

T. C. Adamson, Jr.
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The University of Michigan

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(I) INTRODUCTION

This final technical report is concerned with the tasks specified in the governing contract. They are as follows:

(1) Computer solutions developed during the previous contract year will be used in making a short (one to two minutes) movie film illustrating unsteady flow with a shock wave in a two dimensional symmetric channel; the unsteady behavior will arise as a result of oscillatory pressure pulses introduced in a plenum downstream of the shock wave.

(2) The study of the characteristic time for the formation and collapse of a shock induced separation bubble will be completed.

(3) An attempt will be made to use methods developed previously to develop a criterion for incipient separation induced by an oblique shock wave impinging on a flate plate turbulent boundary layer in transonic and in supersonic flow; the extent to which this analysis will be carried out will depend upon the time remaining after completion of the first two (2) tasks.

The solutions referred to in task (1) are shown in reference 1. They describe the motion of a shock wave and the time and space variations of the flow properties downstream of the shock wave in a nozzle flow when the pressure in the plenum (back pressure) is made to oscillate. One of the more interesting cases for which they are valid is that in which the shock wave moves up to and passes through the nozzle throat, disappearing upstream in the subsonic flow. The solution for

the shock wave velocity given in reference 1, being a first approximation, gives an infinite velocity for the shock wave as it passes through the throat; that is, the solution is singular there as indicated previously[1]. It was decided to analyze the flow in the region of the throat and thus obtain a solution which is uniformly valid throughout the channel, including the throat. This was done and the results included in the solutions used in making the movie referred to in task (1). In addition, a paper covering the derivation of the solutions from which the movie was made was written;[2] it was presented at the AIAA 16th Aerospace Sciences Meeting, Huntsville, Alabama, January, 1978, and has been accepted for publication in the AIAA Journal.

As a result of the extra work done in improving the solutions and preparing the publication, it was not possible to do work of any significance on task (3). As noted in the description of task (3), it was to have been attempted only in the event tasks (1) and (2) were completed before the contract ended.

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[1] Adamson, Jr., T. C. and Liou, M. S., "Unsteady Motion of Shock Waves in Two Dimensional Transonic Channel Flows," Final Technical Report prepared for Naval Air Systems Command, Report UM014534-F, June, 1977.

[2] Adamson, Jr., T. C., Messiter, A. F. and Liou, M. S., "Large Amplitude Shock Wave Motion in Two Dimensional Transonic Channel Flow," AIAA paper no. 78-247, A.I.A.A. 16th Aerospace Sciences Meeting, Huntsville, Alabama, January, 1978.

(II) RESUME OF CASES SHOWN IN COMPUTER MOVIE

The analysis of large amplitude shock wave motion in unsteady transonic flow, contained in reference 1, showed that when conditions are such that oscillations in the back pressure cause the shock to move up to the throat and then disappear upstream, there are two possible subsequent motions. In the first, decreasing back pressure causes a shock wave to form at the throat and then move downstream until the back pressure rises again forcing the shock wave to move back upstream and pass through the throat again; thus, as this process repeats itself, the shock wave oscillates about the throat. In the second, decreasing back pressure again causes the shock wave to form at the throat and move downstream, but it never again reaches the throat, oscillating instead about its steady state position. Each of these cases is illustrated in the computer movie.

The movie was made by showing the desired solution at a given instant of time on a screen and taking a picture of this solution as one frame of the movie; another solution at a given Δt later is then displayed and another frame taken, etc. In this case, the channel wall, shock wave position in the channel, and the instantaneous pressure distribution are all displayed. The exhaust plenum pressure oscillates sinusoidally at 20 Hertz with an amplitude of 2% of the pressure at the throat in each case. In order that the motion of the shock wave and the corresponding changes in pressure distribution can be followed properly, the solutions are displayed at 1/100 real time. In each case, the wall shape of the channel is as follows, where x measures the distance from the throat and is dimensionless with respect to

the channel half width.

$$y_w = \pm (1 - 0.01\, f(x)) \qquad (1a)$$

$$f(x) = 18x^2/13 \qquad x < 1$$

$$= 27(x-2)^4/13 + 48(x-2)^3/13 + 3 \qquad 1 \le x \le 2 \qquad (1b)$$

$$= 3 \qquad x > 2$$

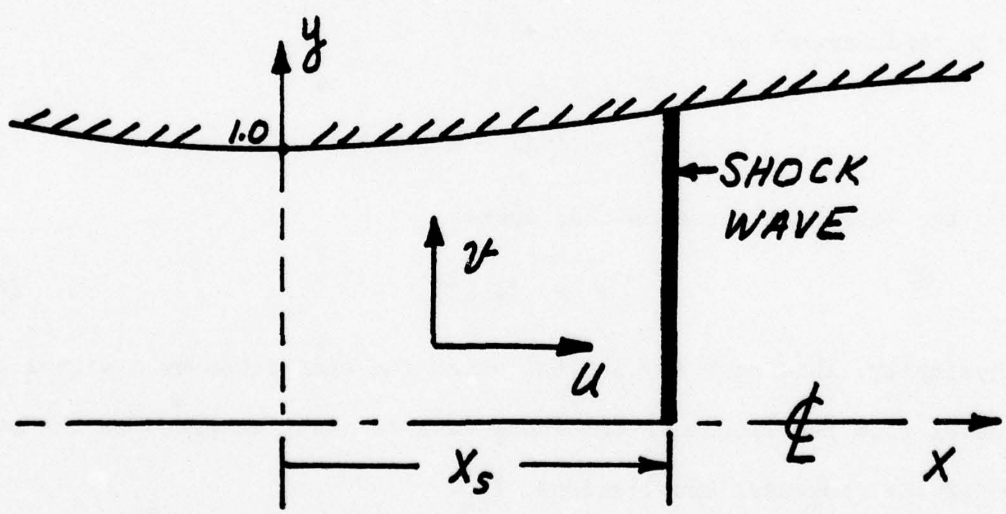
This shape was the result of requiring a parabolic form at the throat and continuous first and second derivatives throughout so no extraneous shock waves would be formed. The only difference between the two cases is the initial, steady state, location of the shock wave. In case I (oscillation about the throat) the steady flow shock wave is located such that the Mach number entering it is $M = 1.08$. In case II, the equivalent Mach number is 1.10. A brief explanation for each case is contained in the film.

(III) DERIVATION OF IMPROVED EQUATION FOR CALCULATING THE VELOCITY OF A SHOCK WAVE AS IT TRAVELS UPSTREAM THROUGH A NOZZLE THROAT

The solutions for unsteady channel flow with a shock wave are given in references 1 and 2. The source of the unsteadiness in the flow is an impressed pressure oscillation in the plenum chamber into which the flow exhausts; these oscillations perturb a steady flow in which a shock wave occurs. The flow upstream of the shock wave is therefore steady. The emphasis in references 1 and 2 is on large scale movement of the shock wave and in particular on those cases where the shock wave actually moves through the throat, disappears upstream, and

forms again at the throat and moves downstream when the back pressure becomes low enough. In the solutions given in these references, there is a singularity at the throat; for example, the solutions for the shock wave velocity give infinite shock velocity at the throat. This is clearly physically impossible. As it turns out it is possible to find corrected solutions valid in an inner region enclosing the throat; these solutions may then be used to construct uniformly valid composite solutions. The inner region and composite solutions are derived in this section.

The following sketch shows the channel and notation used here.



All lengths are made dimensionless with respect to the throat half width, $\bar{\ell}$, and all velocity components with respect to the critical sonic velocity, $\bar{a}^*$. (Overbars denote dimensional quantities.) The slowly varying time regime is considered so that if T is the time made dimensionless with respect to $\bar{\ell}/\bar{a}^*$, then t is defined such that

$$T = \tau t \qquad \tau = \frac{\bar{T}_{ch}}{(\bar{\ell}/\bar{a}^*)} >> 1 \tag{1a,b}$$

where $\bar{T}_{ch}$ is the characteristic (e.g., the period) associated with the impressed oscillations and

$$t = \frac{\bar{T}}{\bar{T}_{ch}} = O(1). \tag{2}$$

The wall shape is arbitrary and given by (where $E << 1$ here replaces ε in references 1 and 2)

$$y_w = \pm (1 - E^2 f(x)) \tag{3}$$

and the case considered is that where

$$\tau = (k\, E^2)^{-1} \tag{4}$$

Physically, this case is that for which the time taken by a signal to travel from the plenum to the shock wave is small compared to the period of the impressed oscillations, $\bar{T}_{ch}$.

The solutions for u and u_s, where u_s is the dimensionless shock wave velocity, are[1,2]

$$u = 1 + Eu_1 + E^2u_2 + \ldots \tag{5a}$$

$$u_1 = \pm \left[\frac{2}{(\gamma+1)} f(x) + C_w\right]^{1/2} \tag{5b}$$

$$u_2 = f'' \frac{y^2}{2} + \zeta^*_{x^*} + h_x \tag{5c}$$

$$h_x = -\frac{1}{6}\left[f'' + (2\gamma-3)u_1^{\,2}\right] + \frac{C_2 + G(t)}{u_1} \tag{5d}$$

$$\zeta^* = \frac{4f''_o}{\pi^3}\left[(\gamma+1)C_u\right]^{1/2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \exp.\ \{-n\pi x^*/[(\gamma+1)C_u]^{1/2}\}$$

$$\cdot \cos(n\pi y) \tag{5e}$$

$$x^* = (x-x_{so})E^{-1/2} \tag{5f}$$

$$u_s = k\, E^2 \frac{d}{dt}(x_{so} + E\, x_{s1} + \ldots) = -E^2 \frac{(\gamma+1)}{4C_u}\left[\frac{2\gamma}{3} C_u^3 + C_{2d} + G(t)\right] + \ldots \tag{5g}$$

where C_w and C_2 are constants of integration, with different values upstream and downstream of the shock wave, and where the plus and minus signs in equation (5b) hold upstream and downstream of the shock wave respectively. The subscripts u and d refer to conditions immediately upstream and downstream of the shock wave respectively, and C_u is the value of u_1 immediately upstream of the shock wave (i.e., eqn. (5b) with the + sign and $C_w = 0$); the term f''_o is used to denote $f''(x_{so})$. The expansion for the shock wave position is $x_s = x_{so} + Ex_{s1} + \ldots$; only the equation for x_{so} is shown and the value for x_{so} is found by integrating equation (5g). Finally, G(t) is proportional to the oscillation in pressure imposed downstream of the shock wave in the plenum;

it is therefore an arbitrary function of time.

The derivation of the improved solution is best illustrated by choosing a given wall shape in the neighborhood of the throat; here, the often used parabolic form is considered:

$$f(x) = ax^2 \tag{6}$$

With this form, it is easily seen that $C_u \propto X_{so}$ so that as $x_{so} \to 0$ $u_s \to \infty$, from equation (5g), unless the bracket $[2\gamma C_u^3/3 + C_{2d} + G(t)]_{x_{so}}$ $= C_{2d} + G(t)$ also is zero at the given instant. As it turns out[1], $C_{2d} + G(t)$ is indeed zero when the shock wave is forming at the throat preparatory to moving downstream; analysis shows that the shock wave velocity there is not singular. However, $C_{2d} + G(t)$ is <u>not</u> zero when the shock wave is moving upstream through the throat and it is for this condition that the following analysis holds.

When the shock wave is very near the throat, the fluid velocity downstream of the shock wave may be written as follows, by using equation (6) in equation (5a) to (5f) and expanding for $x \ll 1$:

$$u = 1 - E\sqrt{\frac{2a}{\gamma+1}}\, x + E^2\Big(a\big(y^2 - \frac{1}{3}\big) - \frac{(2\gamma-3)}{3(\gamma+1)}\, a\, x^2 + \zeta^*_{x^*} - \sqrt{\frac{\gamma+1}{2a}}\,\frac{(C_{2d} + G(t))}{x}\Big) + \ldots \tag{7}$$

Thus, it is seen that as the shock wave moves to the throat and so $x \to 0$ must be considered, the velocity downstream of the shock also is singular for $C_{2d} + G(t) \neq 0$. Moreover, it is seen when $x = O(E^{1/2})$, the first and second order terms become of the same order, so an inner region must be considered, of order $E^{1/2}$ in thickness. In this region,

then, a new coordinate $\tilde{x}$ is defined such that

$$\tilde{x} = x/E^{1/2} \tag{8}$$

and so, equation (7) may be written as follows

$$u = 1 - E^{3/2}\left\{\sqrt{\frac{2a}{\gamma+1}}\,\tilde{x} + \sqrt{\frac{\gamma+1}{2a}}\,(C_{2d} + G(t))/\tilde{x}\right\}$$

$$+ E^2 a\left(y^2 - \frac{1}{3}\right) - E^3\,\frac{(2\gamma-3)}{3(\gamma+1)}\,a\,\tilde{x}^2 + \ldots \tag{9}$$

In the same way one can use equation (5g) to write the corresponding equations for the shock wave velocity:

$$u_s = -E^{3/2}\left(\frac{\gamma+1}{4}\right)\sqrt{\frac{\gamma+1}{2a}}\,(C_{2d} + G(t))/\tilde{x}_s + \ldots \tag{10}$$

Equations (9) and (10) are then the outer solutions to which the inner solutions must match term by term.

From equation (9), it is seen that in the inner region $u = 1 + O(E^{3/2})$. Hence, one can define a velocity potential where $u = \phi_x = E^{-1/2}\phi_{\tilde{x}}$ and $v = \phi_y$ as follows

$$\phi = E^{1/2}\tilde{x} + E^2\tilde{\phi}_1(\tilde{x},y,t) + E^{5/2}\tilde{\phi}_2(\tilde{x},y,t)$$

$$+ E^3\tilde{\phi}_3(\tilde{x},y,t) + \ldots \tag{11}$$

Because the flow is transonic and the shock waves weak, it is possible to define a velocity potential to the desired order.

The wall boundary conditions are found by expanding equation (3) about $\tilde{x} = 0$, writing the resulting equation in terms of $\tilde{x}$, and requiring that the flow be tangent to the walls. Then

$$\tilde{\phi}_{1\tilde{y}}(\tilde{x}, \pm 1, t) = 0 \tag{12a}$$

$$\tilde{\phi}_{2\tilde{y}}(\tilde{x}, \pm 1, t) = \pm 2a\tilde{x} \tag{12b}$$

$$\tilde{\tilde{\phi}}_{3\tilde{y}}(\tilde{x}, \pm 1, t) = 0 \tag{12c}$$

Finally, the jump conditions are written relative to the moving shock wave[3,4]. Since from equation (10) it is seen that $u_s = O(E^{3/2})$ in the inner region, the general expansion for the shock wave velocity and the jump condition are:

$$u_s = E^{3/2}\tilde{u}_{so}(t) + \ldots \tag{13a}$$

$$(u_u - u_s)(u_d - u_s) = 1 - 2\left(\frac{\gamma-1}{\gamma+1}\right)u_s + \ldots \tag{13b}$$

It should be noted that the inner region in question is stationary, enclosing the nozzle throat, and the shock wave moves in it. Hence, unsteady flow equations hold. For example, then, the relationship derived for h_t in the inertial frame in reference 1 is, in the present notation,

$$h_t = \frac{\tilde{a}^2}{\gamma-1} + \frac{u^2+v^2}{2} = \frac{\gamma+1}{2(\gamma-1)} + (u_d - u_u)u_s + \phi_T \tag{14}$$

This is, then, the equation for $\tilde{a}$, the dimensionless speed of sound. If equations (8), (11) and (14) are substituted into the unsteady flow

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[3] Richey, G. K., and Adamson, Jr., T. C., "Analysis of Unsteady Transonic Channel Flow with Shock Waves," AIAA Journal, 14, 1976, pp. 1054-1061.

[4] Chan, J. S.-K., and Adamson, Jr., T. C. "Unsteady Transonic Flows with Shock Waves in an Asymmetric Channel," AIAA Journal, 16, 1978, pp. 377-384.

potential equation[5], linear governing equations for the $\tilde{\phi}_i$ are obtained. These equations are easily integrated and after applying the boundary conditions, equations (12a) - (12c), one finds that downstream of the shock wave, $\tilde{x} > \tilde{x}_s$, or $x > x_s$,

$$\tilde{\phi}_{1\tilde{x}} = - \sqrt{\frac{2a}{(\gamma+1)} \tilde{x}^2 + \tilde{C}_1(t)} \tag{15a}$$

$$\tilde{\phi}_{2\tilde{x}} = a(y^2 - \frac{1}{3}) + \frac{\tilde{M}(t)}{\tilde{\phi}_{1\tilde{x}}} \tag{15b}$$

where $\tilde{C}_1(t)$ and $\tilde{M}(t)$ are functions of integration. These solutions are used to write the inner solutions for u, and the resulting equation is written for $\tilde{x} \gg 1$ and compared with equation (9). For the solutions to match it is found that for $x > x_s$

$$\tilde{C}_1(t) = 2[C_{2d} + G(t)] \tag{16a}$$

$$\tilde{M}(t) = 0 \tag{16b}$$

Upstream of the shock wave the solutions are similar to equations (15); that is, $\tilde{\phi}_{1\tilde{x}} > 0$, and $\tilde{C}_1(t) = \tilde{M}(t) = 0$ since the flow is sonic at the throat. Hence the solutions upstream of the shock wave are simply the expansions of the outer solutions written in inner variables.

With $\tilde{\phi}_{1\tilde{x}}$ known upstream and downstream of the shock wave one can use equations (11) and (13) to show that the shock wave velocity in the inner region is

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[5]Guderley, K. G., The Theory of Transonic Flow, Pergamon Press, Addison-Wesley, 1962, p. 7.

$$\tilde{u}_{so} = -\left(\frac{\gamma+1}{4}\right)\left[\sqrt{\frac{2a}{(\gamma+1)}\tilde{x}_s^2 + \tilde{C}_1(t)} - \sqrt{\frac{2a}{\gamma+1}}\,\tilde{x}_s\right] \qquad (17)$$

It is seen then that for $\tilde{x} >> 1$ and $\tilde{C}_1$ as given in equation (16a), the inner shock wave velocity, equations (13a) and (17), matches with the outer shock wave velocity, equation (10), as it should.

Finally, by adding the inner and outer solutions and subtracting the common terms (those used in matching), one can write composite solutions for u_s and u for $x > x_s$, uniformly valid to order $E^{3/2}$ throughout the channel. Thus,

$$u = 1 + E\left\{u_1 - \sqrt{\frac{2a}{(\gamma+1)}x^2 + 2E(C_{2d} + G(t))} + \sqrt{\frac{2a}{\gamma+1}}\,x\right\}$$

$$+ E^2\left\{\frac{f''}{2}\left(y^2 - \frac{1}{3}\right) - \frac{1}{6}(2\gamma-3)u_1^2 + \delta^*_{x^*}\right.$$

$$\left. + [C_{2d} + G(t)]\left(\frac{1}{u_1} + \sqrt{\frac{\gamma+1}{2a}}\,\frac{1}{x}\right)\right\} + \ldots \qquad x > x_s \qquad (18a)$$

$$u_s = -E\left(\frac{\gamma+1}{4}\right)\left\{\sqrt{\frac{2a}{(\gamma+1)}x_s^2 + 2E(C_{2d} + G(t))} - \sqrt{\frac{2a}{\gamma+1}}\,x_s\right\}$$

$$+ E^2\left(\frac{\gamma+1}{4}\right)\left\{\left(\frac{1}{u_{1u}} - \sqrt{\frac{\gamma+1}{2a}}\,\frac{1}{x_s}\right)(C_{2d} + G(t)) + \frac{2\gamma}{3}u_{1u}^2\right\} + \ldots \qquad (18b)$$

It is seen that as $x \to 0$, u_s and u remain finite, as they should.

It should be noted that the solutions represented by equation (18) are necessary only for $C_{2d} + G(t) \geq 0$, i.e., for the case where the shock wave is approaching the throat from downstream, or when subsonic flow exists throughout the channel.(1) As soon as $C_{2d} + G(t)$ becomes negative, corresponding to the shock wave forming at the throat

and moving downstream,[1] the shock velocity and flow downstream of the shock are finite and no inner solutions are necessary; the outer solutions as presented in reference 1 are valid to the order indicated.

(IV) SHOCK INDUCED SEPARATION IN UNSTEADY FLOW FIELDS

The fundamental motivation for this work is the possibility of adapting for use in unsteady channel or nozzle flows the known steady flow solutions for the interaction between a shock wave and a boundary layer.[6] In a previous study,[1] it was shown that the partial time derivatives could be neglected compared to spatial derivatives in the conservation equations (e.g., $\partial u/\partial t \ll u\partial u/\partial x$) in many cases of technical interest. This indicates that in the terminology generally used for unsteady flows, these problems of interest fall in the so called slowly varying, or quasi steady, time regime where the characteristic time associated with the flow disturbance is large compared with the fluid residence time in the channel. Since time derivatives do not appear in the governing equations, the unsteady flow problem may be considered as a series of steady state solutions each with a different set of boundary conditions. It should be noted, however, that the proper boundary conditions in this case do not always coincide with those used in solutions to steady state flow problems; this point will be dealt with in the present context later.

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[6] Liou, M. S., "Asymptotic Analysis of Interaction Between a Normal Shock Wave and a Turbulent Boundary Layer in Transonic Flow," Ph.D. Thesis, University of Michigan, 1977.

In Meiers'[7] experiments involving shock induced oscillations in flow in a nozzle, it was found that at a certain point in the cycle a shock occurred, strong enough to cause the flow to separate or to move the point of detachment of already separated flow. Then, as the shock moved upstream, the separation point moved with it. Meier implies that the shock induced separation is a necessary part of the observed oscillatory instability. However, it can be seen in the experimental pictures that the distance between the shock wave and the separation point varies significantly with time and grows to be of the order of the channel width. If this behavior is indeed being induced by the shock wave boundary layer interaction it is due to unsteady effects because such large distances are not found in typical steady flow interactions between shock waves and turbulent boundary layers in transonic flow.[6,8] It was first believed that the unsteady effects were strong enough to cause local accelerations to be important, thus changing the problem completely from that in the steady flow case. However, as mentioned above, this was shown not to be the case for problems of interest (including Meiers' experiments) so this possibility must be ruled out. On the other hand, it is clear that even though the solution is quasi steady, the shock position and distance from the

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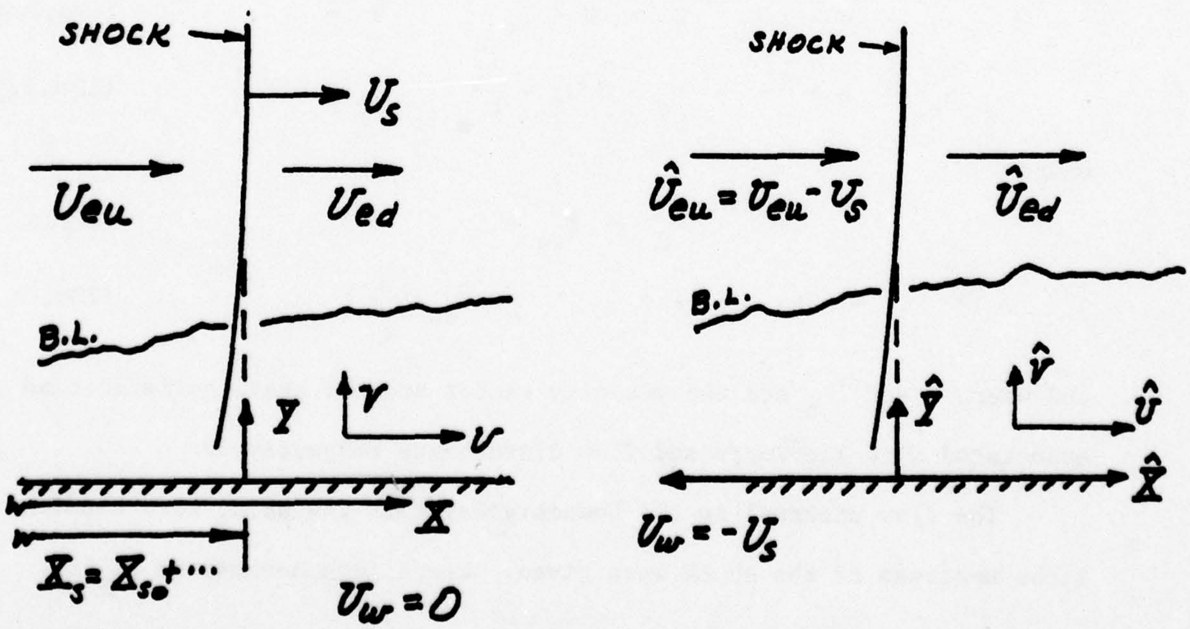
[7] Meier, G.E.A., "Shock Induced Flow Oscillations," AGARD-CP-168, Flow Separation, 1975, pp. 29-1 to 29-9.

[8] Kooi, J. W., "Data Report of a Transonic Shock-Wave Boundary Layer Interaction Experiment at a Mach Number of 1.44 and 1.46," N.L.R. Memorandum AC-76-018, The Netherlands, November, 1977.

shock to the separation point are functions of time and it is of interest to check to see if the predicted values of these quantities can grow to the size indicated in the experiments. It is the purpose of this section to use known steady flow solutions[6] as a basis for such calculations and to provide an estimate of the time for the formation and collapse of a shock wave induced separation bubble. The results are used to draw conclusions concerning the experimental results.

We consider a shock wave moving over a turbulent boundary layer on a flat plate in transonic, two dimensional flow. The effects of the curvature of the channel, while certainly important insofar as quantitative results are concerned, are not important insofar as orders of magnitude are concerned and are therefore neglected. The flow upstream of the shock is steady, just as in the channel flow. We then take the viewpoint of an observer moving with the shock wave. The absolute and relative points of view and the associated notation are shown in the following sketch: $X_s = X_{so} + \ldots$ is the shock wave location.



Thus, the caret is used to indicate variables in the relative frame. Subscripts e, w, and s refer to external flow, wall and shock wave conditions respectively, while the subscripts u and d refer to conditions immediately upstream and downstream of the shock respectively. Lengths are made dimensionless with respect to a characteristic length $\bar{\ell}$, e.g., the channel half width at the minimum area, and velocity components are made dimensionless with respect to $\bar{a}^*$, the critical sonic velocity in the flow external to the boundary layer. (Overbars denote dimensional quantities.) The enthalpy and stagnation enthalpy are referred to $\bar{a}^{*2}$, and the remaining thermodynamic quantities are referred to their critical values in the core channel flow. The time, T, is referred to $\bar{\ell}/\bar{a}^*$.

The relationships between the absolute and relative frame quantities are as follows.

$$\hat{X} = X - X_{so} \qquad \hat{Y} = Y \qquad \hat{T} = T \qquad \hat{t} = t \tag{19a,b,c,d}$$

$$\vec{\hat{q}} = \vec{q} - \vec{i}\, U_S \qquad \hat{U} = U - U_S \qquad \hat{V} = V \tag{19e,f,g}$$

$$\hat{h}_t = h + \frac{\hat{q}^2}{2} = h_t - \hat{U}\, U_S - \frac{U_S^2}{2} \qquad h_t = h + \frac{q^2}{2} \tag{19h,i,j}$$

where

$$U_S = \dot{X}_{so} + \ldots \tag{20a}$$

$$\hat{T} = \tau t \qquad \tau = \bar{T}_{ch}/(\frac{\bar{\ell}}{\bar{a}^*}) \tag{20b,c}$$

and where $\vec{q}$ and $\bar{T}_{ch}$ are the velocity vector and the characteristic time associated with the impressed flow disturbance respectively.

The flow external to the boundary layer is inviscid, with conditions upstream of the shock wave given. Conditions downstream of the

wave are then found by application of the jump conditions across a normal shock wave in the relative system. The relative stagnation enthalpy, which is unchanged across the shock wave, may be evaluated immediately upstream of the shock using equations (19i) and (19f) and the known velocity upstream of the shock. Thus

$$\hat{h}_{tu} = h_{t_u} - \hat{U}_{eu}U_S - \frac{U_S^2}{2} = \frac{\gamma+1}{(\gamma-1)} - U_{eu}U_S + \frac{U_S^2}{2} \tag{21}$$

where since the flow upstream of the wave is steady, $h_{tu} = (\gamma+1)/2(\gamma-1)$; however, because the shock position changes with time, $U_{eu} = U_{eu}(T)$ and since $U_S = U_S(T)$, $\hat{h}_{tu}$ is a function of time. Then, using the equations for the jump conditions across a normal shock wave, one can show that the equivalent Prandtl relationship is,

$$\hat{U}_{ed} = \frac{1}{\hat{U}_{eu}} [1 - \frac{2(\gamma-1)}{(\gamma+1)} (\hat{U}_{eu}U_S + \frac{U_S^2}{2})] \tag{22}$$

Now, for the case to be considered here, conditions for the channel core flow coincide with case (2) of reference 1; that is, the flow is transonic and the shock velocity is $O[U_{eu} - 1)^2]$. The change in shock wave position is of order unity. Although the solutions given in this reference are for the case where an oscillating back pressure causes the shock wave motion, this is not a significant point for the present analysis. It is only the relative order of parameters and the accompanying form of the expansions that are necessary here, and these are independent of the form of forcing function for the oscillation.

In order to simplify the notation used in the present analysis,

the flow relative to the shock wave, external to the boundary layer is written as $\hat{U}_{eu} = 1 + \hat{\varepsilon}(\hat{T})$, similar to the notation used in the analysis of the interaction between a shock wave and a boundary layer in steady flow.[6] The relationship between $\hat{\varepsilon}$ and the solutions for unsteady channel flow[1], where these solutions are expanded in powers of E, say, (here E replaces ε used in reference 1) is as follows:

$$U_{eu} = 1 + E\, u_{e1}(X_{so}) + E^2 u_{e2}(X_{so},0,T) + \dots \tag{23a}$$

$$\hat{U}_{eu} = 1 + \hat{\varepsilon}(T) = \hat{U}_{eu} - U_S \tag{23b}$$

That is, in general, the inviscid flow solution for the channel is such that

$$U(X,Y,T) = 1 + E\, u_1(X) + E^2 u_2(X,Y,T) + \dots$$

and so, for example, $U_u = U_u(X_{so},Y,T)$; then equations (21) and (22) hold in general across the wave with U_u and U_d replacing U_{eu} and U_{ed}. Here, since the velocity immediately external to the boundary layer at the shock wave is desired, U_{eu} is the value of U evaluated at the wall (Y = 0 in the present notation) as well as at $X = X_{so}(T)$. U_{sh} is the absolute velocity of the shock wave and is of order E^2. That is, for the case under consideration where $\tau = (k\, E^2)^{-1}$

$$U_S = \frac{dX_{so}}{dT} + \dots = k\, E^2 \frac{dX_{so}}{dt} + \dots \tag{24a}$$

$$\frac{dX_{so}}{dt} = O(1) \tag{24b}$$

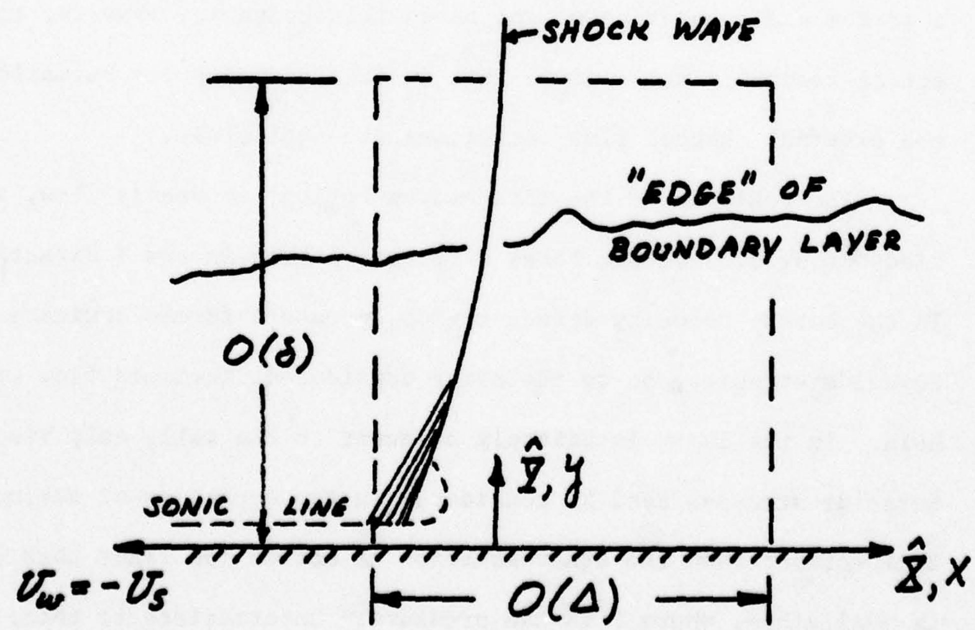
Thus $\hat{\varepsilon}$ is a function of time defined in terms of E, U_S, and $u_1, u_2, \dots$ evaluated at $X = X_{so}$ and Y = 0.

If equation (23b) and (24a) are substituted into equation (22) and only those terms which give contributions up to $O(\hat{\varepsilon}^2)$ are retained, then it is seen that

$$\hat{U}_{ed} = \frac{1}{\hat{U}_{eu}} \left(1 - 2\left(\frac{\gamma-1}{\gamma+1}\right)U_S\right) + \dots \tag{25}$$

Now, in the steady flow case[6], the result is $U_{ed} = U_{eu}^{-1}$, with $U_{eu} = 1 + \varepsilon$. Hence, in this unsteady case, there is an additional second order term in the flow velocity downstream of the shock wave, external to the boundary layer. Similar terms would be found in the temperature, density, and pressure in the external flow.

The interaction region, as seen in the relative coordinate system, is shown in the following sketch. In the $\hat{X}$ direction



its extent is $O(\Delta)$ while in the Y direction it is of the order of the boundary layer thickness, δ. Thus, new variables x and y are defined as follows:[6]

$$\frac{\hat{Y}}{L} = \delta y; \quad \frac{\hat{X}}{L} = \Delta x; \quad \delta = \frac{\bar{\delta}}{\bar{L}}; \quad \Delta = (\gamma+1)^{1/2}\hat{\varepsilon}^{1/2}\delta \qquad (26a,b,c,d)$$

With velocity components $\hat{U} = \hat{U}(x,y)$ and $\hat{V} = \hat{V}(x,y)$ in the interaction region. In equation (26), $L = \bar{L}/\bar{\ell}$ is the dimensionless length of the boundary layer and is thus equal to X_{so} plus the distance from the channel throat to the beginning of the boundary layer; L is a function of time. Because $\Delta \ll 1$ and is in fact small compared to the extent of the inner region enclosing the shock wave in the channel flow solutions, the velocity in the flow external to the boundary layer in the interaction region is given to lowest order by the values $\hat{U}_{eu}$ and $\hat{U}_{ed}$, upstream and downstream of the shock respectively. That is, the interaction region is thin enough that in this distance the variations in the external channel flow velocities are negligible.

The analysis of the interaction region, in steady flow, is carried out by considering three different layers in the $\hat{Y}$ direction.[6] In the outer, velocity defect region, pressure forces dominate over Reynolds stresses, so to the order considered, [illegible]id flow equations hold. In the layer immediately adjacent t[illegible], only viscous and Reynolds stresses need be considered in the equations of motion in the $\hat{X}$ direction; from the other equation of motion one finds that $\partial P/\partial Y$ is negligible, where P is the pressure. Intermediate to these two layers is a layer in which inertia, pressure gradient, and Reynolds shear

stress terms dominate one equation of motion while the other again shows $\partial P/\partial Y$ negligible to the order retained. Throughout the interaction region, the stagnation enthalpy remains constant since the wall is taken to be adiabatic and the turbulent and laminar Prandtl numbers are assumed to be unity. The solutions in the outer, inviscid flow, layer may be found independently of the inner two layers, for unseparated flow, and allow calculation of the wall pressure distribution. With this pressure distribution known, the solutions in the two inner layers may be used to calculate the wall shear stress distribution, τ_w. The question now is whether these solutions or some modified form of them hold in the present case.

The forms of the conservation equations which hold in the relative coordinate system are derived from the general unsteady conservation equations by using the following transformations

$$\frac{\partial}{\partial T} = \frac{\partial}{\partial \hat{T}} - U_S \frac{\partial}{\partial \hat{X}} \qquad \frac{\partial}{\partial X} = \frac{\partial}{\partial \hat{X}} \qquad \frac{\partial}{\partial Y} = \frac{\partial}{\partial \hat{Y}} \tag{27a,b,c}$$

The equations which hold in the turbulent boundary layer in the relative system are then,

$$\frac{\partial \rho}{\partial \hat{T}} + \nabla \cdot \rho\hat{\vec{q}} = 0 \tag{28a}$$

$$\rho \frac{\partial \hat{\vec{q}}}{\partial \hat{T}} + (\rho\hat{\vec{q}} \cdot \hat{\nabla})\hat{\vec{q}} + \vec{i}\,\dot{U}_{sh}\rho = -\frac{1}{\gamma}\hat{\nabla} P + (\quad) \tag{28b}$$

$$\rho \frac{\partial \hat{h}_t}{\partial \hat{T}} + (\rho\hat{\vec{q}} \cdot \hat{\nabla})\hat{h}_t + \hat{U}\,\dot{U}_{sh}\rho = \frac{1}{\gamma}\frac{\partial P}{\partial \hat{T}} + (\quad) \tag{28c}$$

where the empty brackets in equations (28b) and (28c) represent viscous and Reynolds stress terms. Only those terms which involve differences between those important in the present problem and those important in the steady flow case are shown here; that is, since $\partial\hat{U}/\partial\hat{Y} = \partial U/\partial Y$, Reynolds and viscous stress terms have the same form in either case. Also, it should be noted that in order to consider unsteady turbulent flow, it is possible to consider only those time scales which are large compared to the time scale necessary to define a time average in the turbulent flow.

It has been shown in reference 6 that the major contributions to the expression for the shear stress at the wall, τ_w, are found by consideration of the flow downstream of the shock wave. This is due to the fact that for shock waves strong enough to cause separation, the upstream influence of the interaction is confined to a region exponentially small, in asymptotic terms, compared to the boundary layer thickness. Hence, it is sufficient to consider the undisturbed boundary layer flow entering a shock wave which extends deep into the boundary layer, causing a large variation in wall pressure; as the shock wave is approached from downstream, a singularity is found. The net effect of this is that in the three aforementioned layers in which solutions must be found, one writes the solutions in terms of perturbations from the post shock conditions in the external flow. The same procedure is used here.

In both the outer inviscid flow layer and the Reynolds stress sublayer (intermediate layer), then, the velocity components have

expansions of the following form in the steady flow analysis

$$U = U_{ed} + u_T u_1(x,y) + \ldots = 1 - \varepsilon + \ldots + u_T u_1(x,y) + \varepsilon u_T u_2(x,y), \ldots$$

$$U = \varepsilon^{1/2} u_T v_1(x,y) + \varepsilon^{3/2} u_T v_2(x,y) + \ldots$$

with similar expressions for the pressure, density, and temperature. Here, u_T is the dimensionless friction velocity, where $u_T = O(\delta)$, and $u_T \ll \varepsilon \ll 1$. Again, $U_{ed} = U_{eu}{}^{-1}$ in steady flow. The comparable expression in the present case is, then,

$$\hat{U} = \hat{U}_{ed} + u_T \hat{u}_1(x,y,t) + \ldots = 1 - \hat{\varepsilon}(t) + \ldots + u_T \hat{u}_1(x,y,t) + + u_T \varepsilon \hat{u}_2(x,y,t) + \ldots \quad (29)$$

where equation (25) has been used for $\hat{U}_{ed}$. A similar expression holds for P. The equivalent expansion for V is

$$\hat{V} = \hat{\varepsilon}^{1/2} u_T \hat{v}_1(x,y,t) + \hat{\varepsilon}^{3/2} u_T \hat{v}_2(x,y,t) + \ldots \quad (30)$$

Now, if equations (20b), (26), (29), and (30) are substituted into the X component of equation (28b), for example, the result is, for $\tau = (k\,E^2)^{-1} = [k(\frac{\hat{\varepsilon}}{u_{e1}} + \ldots)^2]^{-1}$, and $\rho = 1 + \hat{\varepsilon}(t) + \ldots$,

$$\Big\{ - k(\frac{\hat{\varepsilon}}{u_{e1}})^2 \frac{\partial \hat{\varepsilon}}{\partial t} + \ldots + \frac{u_T}{\Delta}(\frac{\partial \hat{u}_1}{\partial x} + \hat{\varepsilon}\frac{\partial \hat{u}_2}{\partial x} + \ldots)(1 - \hat{\varepsilon} + \ldots)$$

$$+ \varepsilon^{1/2}\frac{u_T}{\delta}(\frac{\partial \hat{v}_1}{\partial y} + \varepsilon \frac{\partial \hat{v}_2}{\partial y} + \ldots) + \ldots \Big\} (1 + \hat{\varepsilon}(t) + \ldots)$$

$$+ k^2(\frac{\hat{\varepsilon}}{u_{e1}})^4 \frac{d^2 X_{so}}{dt^2} + \ldots = -\frac{1}{\gamma}\frac{u_T}{\Delta}(\frac{\partial P_1}{\partial x} + \varepsilon \frac{\partial P_2}{\partial x} \ldots) + (\quad) \quad (31)$$

Now, since $\Delta = O(u_T \hat{\varepsilon}^{1/2})$ and $\delta = O(u_T)$, it is seen that to and

including second order terms, the terms involving time derivatives are small compared to those involving space derivatives. Hence, to the order considered, this equation has the same form as that used in the steady flow solution. Similar calculations result in the same conclusion for the Y momentum and the continuity equations. In the layer adjacent to the wall the density expansion is $\rho = \rho_w(T) + u_T\tilde{\rho}_1 + \ldots$ with similar expansions for the temperature and pressure, and $\hat{U} = u_T\tilde{u}_1 \ldots - U_{sh}$. Since the only difference from the expansions in the other layers is that U is smaller (of higher order) the conclusion is again that equations similar to those used in the steady flow case are found. The energy equations gives

$$\hat{h}_t = \hat{h}_{tu} = \frac{\gamma+1}{(\gamma-1)} - U_{sh} + \ldots \qquad (32a,b)$$

throughout the boundary layer, where equations (21) and (29) have been used to write h_{tu} to order $\hat{\varepsilon}^2$ in equation (32b). Again, U_S is a function of time alone.

The interaction problem becomes, then, a problem with governing equations which have essentially the same form as those in the steady flow problem. The only change, resulting from the fact that the stagnation enthalpy is a function of time rather than a constant, is the appearance in the expansion for the temperature of a term which is a function of time alone and $O(\hat{\varepsilon}^2)$. However, the boundary conditions are different in that $\hat{U} = -U_S$ at the wall, and in the flow external to the boundary layer, the boundary condition is given by equations (25) rather than the steady flow condition, $U_{ed} = (U_{eu})^{-1}$. At each boundary, then there is an added term in the velocity, of order $\hat{\varepsilon}^2$, which is a

function of time alone. Because the wall shear stress term, $\tau_w = \text{constant}(\hat{\mu}\ \partial U/\partial Y)_w$ is found by matching solutions for U throughout the three layers in the interaction region[6] and since these solutions differ from their steady flow counterparts by terms of $O(\hat{\varepsilon}^2)$, it is seen that at most, the expression for the present unsteady flow case will differ from that found for the steady flow case by a term which is a function of time alone and which is of order $\hat{\varepsilon}^2$. Since this extra term is not critical insofar as this analysis is concerned, it has not been derived in detail; instead, it is written as $\hat{\varepsilon}^2 F(t)$. Finally, then, by adding this term to equation 5.79 in reference 6, the steady flow solution, one finds the following equation for τ_w:

$$\tau_w(x) = 1 + a\,\hat{\varepsilon} + [\tfrac{1}{2} a(a-1) + F(t)]\hat{\varepsilon}^2 + \ldots\ u_\tau\left(-\frac{a}{2\gamma} P_1(x)\right)$$

$$+ \hat{\varepsilon} u_\tau (\ln \Delta)\frac{4}{\tilde{C}_o}\left(\frac{1}{\tilde{C}_o} + \gamma + m\right) + \hat{\varepsilon}\, u_\tau \left\{ \frac{a}{2\gamma} P_{11}(x)\right.$$

$$+ 2\frac{\sqrt{T}_{eu}}{\tilde{C}_o}\left(3\gamma + m - \frac{3}{\tilde{C}_o}\right)\left(-\frac{P_1}{\gamma}(x)\right)$$

$$\left. + 4\frac{\sqrt{T}_{eu}}{\tilde{C}_o}\,\alpha\left(\frac{\sqrt{T}_{eu}}{\tilde{C}_o} + \gamma + m\right)\left(\ln x - \gamma_e + \ln\frac{2}{\alpha}\right)\right\}$$

$$+ \ldots \qquad (33)$$

where

$$a = -4\left(\frac{\sqrt{T}_{eu}}{\tilde{C}_o} - \frac{\gamma}{2}\right) \qquad (34a)$$

$$\tilde{T}_{eu} = \frac{\gamma+1}{2} - \left(\frac{\gamma-1}{2}\right)U_{eu}^2 = 1 - \left(\frac{\gamma-1}{2}\right)(U_{eu}^2-1) \tag{34b}$$

$$\tilde{C}_o = \sqrt{\frac{2\tilde{T}_{eu}}{\gamma-1}}\,\sin^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}} \tag{34c}$$

$$\alpha = (0.41)^{-1} = \text{inverse of Kármán constant} \tag{34d}$$

$$\Delta = [(\gamma+1)\hat{\varepsilon}]^{1/2}u_\tau \tag{34e}$$

$$m = -(\gamma-1)\hat{U}_{eu}/2\,\tilde{T}_{eu} \tag{34f}$$

$$P_1(x) = \frac{2\gamma x}{\pi}\int_{-\infty}^{\infty}\frac{u_{01}(\eta)}{(x^2+\eta^2)}\,d\eta \tag{34g}$$

$$P_{11}(x) = (2\gamma-1)P_1(x) + \frac{1}{2}(\gamma + \frac{1}{2})x\,\frac{\partial P_1}{\partial x} \tag{34h}$$

In equations (34), $\tilde{T}_{eu}$ is the dimensionless temperature external to the boundary layer immediately upstream of the shock wave and is, therefore, a function of time; γ is the ratio of specific heats; and u_{01} is the variable part of the velocity of the flow in the velocity defect layer in the undisturbed boundary layer. $(U = U_{eu} + \hat{u}_\tau u_{01} + \ldots)$. As shown, for example in reference 6, a corresponding incompressible profile such as Coles'(9) with the proper coefficients can be used here. The value for u_τ is found from the following equation:

$$u_\tau = \tilde{C}_o/\alpha \ln Re + \ldots \tag{35a}$$

[9] Coles, D., "The Law of the Wake in the Turbulent Boundary Layer," Journal of Fluid Mechanics, vol. 1, 1956, pp. 191-226.

$$Re = \left(\frac{\bar{\rho}^*\bar{a}^*}{\bar{\mu}^*}\right)_{eu} \bar{L} \qquad (35b)$$

Typical variations of τ_w vs x for the steady case[6] (i.e., X_{so} = constant, $U_{sh} = 0$) are shown in figure 1. This corresponds to a plot of $\tau_w - \hat{\varepsilon}^2F(t)$ vs x in the present case, at a given time and thus for a given $\hat{\varepsilon}$. It is seen that τ_w goes through a minimum. Although the solution given in equation (33) is an asymptotic expansion and in mathematical terms is thus valid only when each succeeding term is small compared to the previous term, it has been shown[6] that the use of this expression to predict incipient separation (i.e., $\tau_w = 0$ at $\partial \tau_w/\partial_x = 0$) gives results which appear to agree with the limited experimental results available. Here we assume that the solutions give proper trends also for separated flows as long as the separation bubble is small enough. Plots of $\tau_w - \hat{\varepsilon}^2F(t)$ vs x at incipient separation and when the flow has separated to form a small bubble are shown in figures 2 and 3 respectively.[6] The point to note is that the distance from the shock wave ($x \cong 0$) to the point of separation is less than or equal to the distance to the minimum point of τ_w. Finally, the inclusion of the $\hat{\varepsilon}^2F(t)$ terms will not change these graphs significantly, since $\hat{\varepsilon}$ is at most 0.25.

It is seen, from figures 2 and 3, that as the Mach number (and therefore $\hat{\varepsilon}$) of the flow entering the shock wave increases at constant Reynolds number, the distance from the shock wave to the separation point decreases; i.e., the maximum distance occurs at incipient separation (figure 2). Furthermore, it is easily shown[6] and in fact

illustrated in figure 1 that as the Reynolds number increases (u_τ decreases) the value of $(\tau_w)_{min}$ increases. Thus, for separated flow, the size of the bubble decreases as Reynolds number increases; however, it should be noted that very large changes in Reynolds number cause only very small changes in $(\tau_w)_{min}$ and thus in bubble size. These results allow the following interpretation of the flow in the present unsteady channel flow problem. As the shock wave moves upstream or downstream both the local boundary layer thickness upstream of the shock (or equivalently the local Reynolds number) and the local Mach number of the flow into the shock wave vary with time. Since $(\tau_w)_{min}$ is a function of Reynolds number and since the distance from the shock wave to the separation point varies whenever $(\tau_w)_{min}$ varies, in separated flow, it follows that as the Reynolds number changes the distance from the shock to the separation point changes also. However, since $(\tau_w)_{min}$ is relatively insensitive to large changes in Reynolds number, these variations will be neglected compared to those caused by changes in Mach number. Furthermore, because the shock velocity is of order $\hat{\varepsilon}^2$ and changes in the absolute velocity due to changes in position are of order $\hat{\varepsilon}$, the shock wave velocity may be neglected in assessing the most important effects. Finally, then, the most important parameter in ascertaining the instantaneous distance from the shock wave to the separation point is $\hat{\varepsilon}$, which measures the value of the instantaneous absolute Mach number immediately upstream of the given shock wave position.

If one were to consider an unsteady nozzle flow with steady flow

upstream of the shock wave and with no separation other than that caused by the shock wave, the following sequence of events would be predicted by the above analysis. As the shock wave moves downstream ($\hat{\varepsilon}$ increases) separation occurs at some position and the bubble then proceeds to grow. As soon as the shock reverses position and begins to move upstream, the bubble begins to decrease in size and finally disappears. Hence, the time characteristic of the foundation and collapse of the bubble is the characteristic time of the shock wave motion itself. This is a direct result, again, of the fact that so called quasi-steady solutions hold. The most important point to be noted here is that throughout this sequence of events, the distance from the shock wave to the separation point, say x_{sep}, which is always less than or equal to the distance to the position of $(\tau_w)_{min}$, is such that $x_{sep} = O(1)$. It does not become very large for Mach numbers and Reynolds numbers of interest. Hence,

$$\hat{X}_{sep} = X_{sep} - X_{so} = X_{sep} L \Delta = O(\Delta) \tag{36}$$

for $X_{so} = O(1)$ and $L = O(1)$, the case considered here. Finally, then,

$$\frac{\hat{X}_{sep}}{L} << 1 \tag{37}$$

That is, measured in terms of the distance from the initiation of the boundary layer to the shock wave, the distance from the shock wave to the separation point is small. Although this result was deduced from a solution which holds only for small bubbles (at most of order δ in extent) there is no reason to suppose that the first point of the

bubble (i.e., the separation point) will change position by orders of magnitude because the tail of the bubble moves downstream. Indeed, it appears that as the shock wave becomes stronger, the separation point moves upstream relative to the shock wave, and moves upstream of the normal shock wave as a lambda shock is formed; still the distance between the normal shock wave and the separation point appears to be at most of the order of the boundary layer thickness.[10]

Although the above analysis has been carried out for the case where $\tau = O(E^{-2})$ and $U_s = O(E^2)$, it is seen that the same general results would hold for the case $\tau = O(E^{-1})$ and $U_s = O(E)$, again with variations of shock positions of O(1). That is, shock wave velocities greater by an order of magnitude could be considered also, corresponding to case (1) of reference 1, but with large amplitude shock wave motions. In this event, it can still be shown that in the relative frame of reference, the time derivatives may be neglected to the desired order, and the stagnation enthalpy is given by equations (32a,b). The boundary conditions again differ from those used in the steady flow case by terms involving U_s. Hence, the result for τ_w again is that given by the steady state analysis except for the inclusion of a term involving U_s which is now of order $\hat{\varepsilon}$ rather than $\hat{\varepsilon}^2$. The shock wave velocity can no longer be ignored to lowest order since it is $O(E) = O(\hat{\varepsilon})$, but this leads only to changes in numerical values, not in order. Evidently, then, the same conclusions are reached for this case as the

[10] Vidal, R. J., Wittliff, C. E., Catlin, P. A. and Sheen, B. H., "Reynolds Number Effects on the Shock Wave-Turbulent Boundary Layer Interaction at Transonic Speeds," AIAA paper no. 73-661, AIAA 6th Fluid and Plasma Dynamics Conference, Palm Springs, California, July, 1973.

one analyzed.

In summary, the conclusions reached from the specific analysis carried out here are as follows:

Under the conditions of slowly varying transonic flow in the interaction between a turbulent boundary layer and an oscillating shock wave:

(a) The dimensionless (with respect to the length of the boundary layer) distance from the foot of the shock wave to the point of separation is $O[(U_{eu}-1)^{1/2}\delta]$ where δ is the order of the local boundary layer thickness (dimensionless with respect to the length of the boundary layer) and U_{eu} is the velocity (dimensionless with respect to the external flow critical velocity) of the flow external to the boundary layer, both measured immediately upstream of the shock wave.

(b) The time characteristic of the formation and collapse of the separation bubble is the same order as that associated with the oscillation of the shock wave.

The most significant difference between the flow outlined above and that studied experimentally by Meier(7) is that in Meier's flow, separation was always present in the subsonic part of the nozzle flow, whether or not a shock wave was present. Thus, the separation bubble was modified by but not caused by the shock wave. Furthermore, the separated region was large enough that the resulting core flow formed an asymmetric rather than a symmetric channel flow. Nevertheless, the conclusion concerning the order of the distance from the shock wave to

the separation point is valid. It is independent of both the exact numerical value of the pressure downstream of the thin interaction region and the curvature of the flow external to the boundary layer as long as the flow is transonic as specified.

The above analysis suggests that in Meier's[7] experiments, shock induced separation may be playing a significant part in the observed oscillatory instability only when the separation point is very close to the shock wave (within a distance of order of the boundary layer thickness just upstream of the shock wave). Large scale (of order of the nozzle throat width) oscillations of the distance between the shock wave and separation point evidently are not found in unsteady shock induced separation under quasi-static flow conditions. Therefore, under *these conditions*, it would certainly appear that another mechanism, is the source of the observed oscillatory instability.

Recently, experiments similar to Meier's[7], but more detailed, have been carried out by Sajben, Kroutil,and Chen[11] on a flow exhibiting oscillatory instabilities. They identify several flow regimes including subsonic flow without separation, subsonic flow with separation, supercritical flows with shock waves separated by large distances (of order of the nozzle throat) from the separation point, and supercritical flows with shock waves immediately adjacent to the separated

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[11] Sajben, M., Kroutil, J. C. and Chen, C. P., "Unsteady Transonic Flow in a Two Dimensional Diffuser," McDonnell Douglas Research Laboratories Report no. MDRL 77-12, 1977. (Presented at AGARD Fluid Dynamics Panel Symposium on Unsteady Aerodynamics, Ottawa, Canada, September, 1977.

region. In the last mentioned case, the separation point suddenly moved upstream to the shock foot and the separation bubble nearly doubled in length. The authors concluded that what had been pressure gradient-induced separation in the subsonic flow had changed to shock-induced separation. The present work simply adds analytical confirmation to that conclusion. Evidently, the main effect of the change is the change in bubble size. It is interesting to note that these authors also concluded that it should be possible to use a quasi-steady analysis to study the shock wave boundary layer interaction.

Apparently, then, contrary to the implications in Meier's[7] work, shock-induced separation is not a necessary mechanism in the formation of oscillatory instabilities in nozzle flows, although it may be an important factor in the process. Thus, in the experimental work cited, it appears that when supersonic flow occurs in the nozzle, the oscillations are linked to the existence of a separated region of flow but this separated flow bubble is essentially the result of the adverse pressure gradient in the subsonic part of the flow. Evidently, the oscillations of the shock wave position and the bubble size are connected through the unsteady pressure field downstream of the shock wave such that phase lags between the shock wave motion and rate of change of size of the separated flow region allow an undamped oscillation to exist, this interaction being modified but not changed in principle when shock-induced separation occurs.

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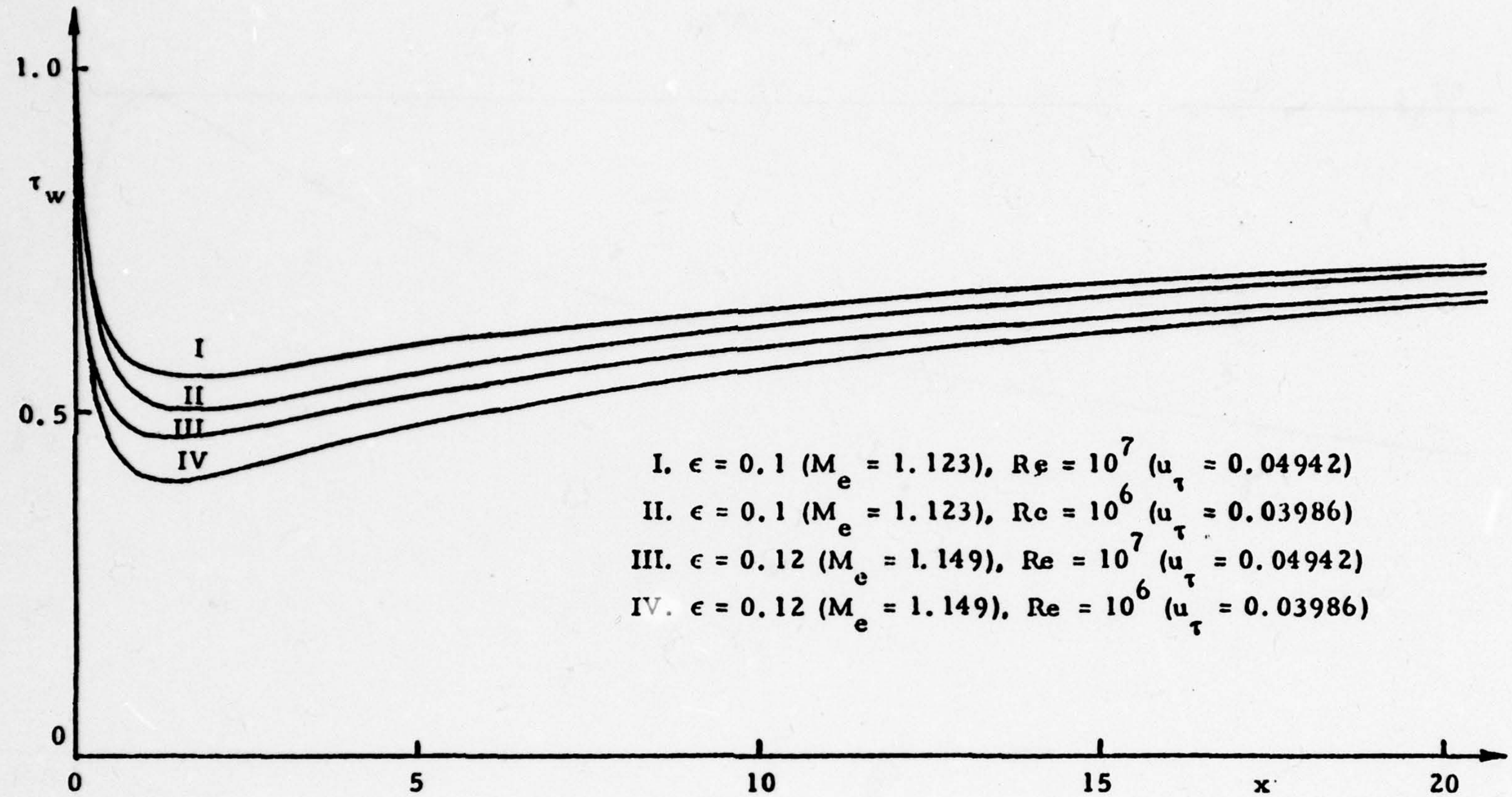

Figure 1

Shear stress distributions at the wall in the outer region for various M_e and Re (from Ref. 6)

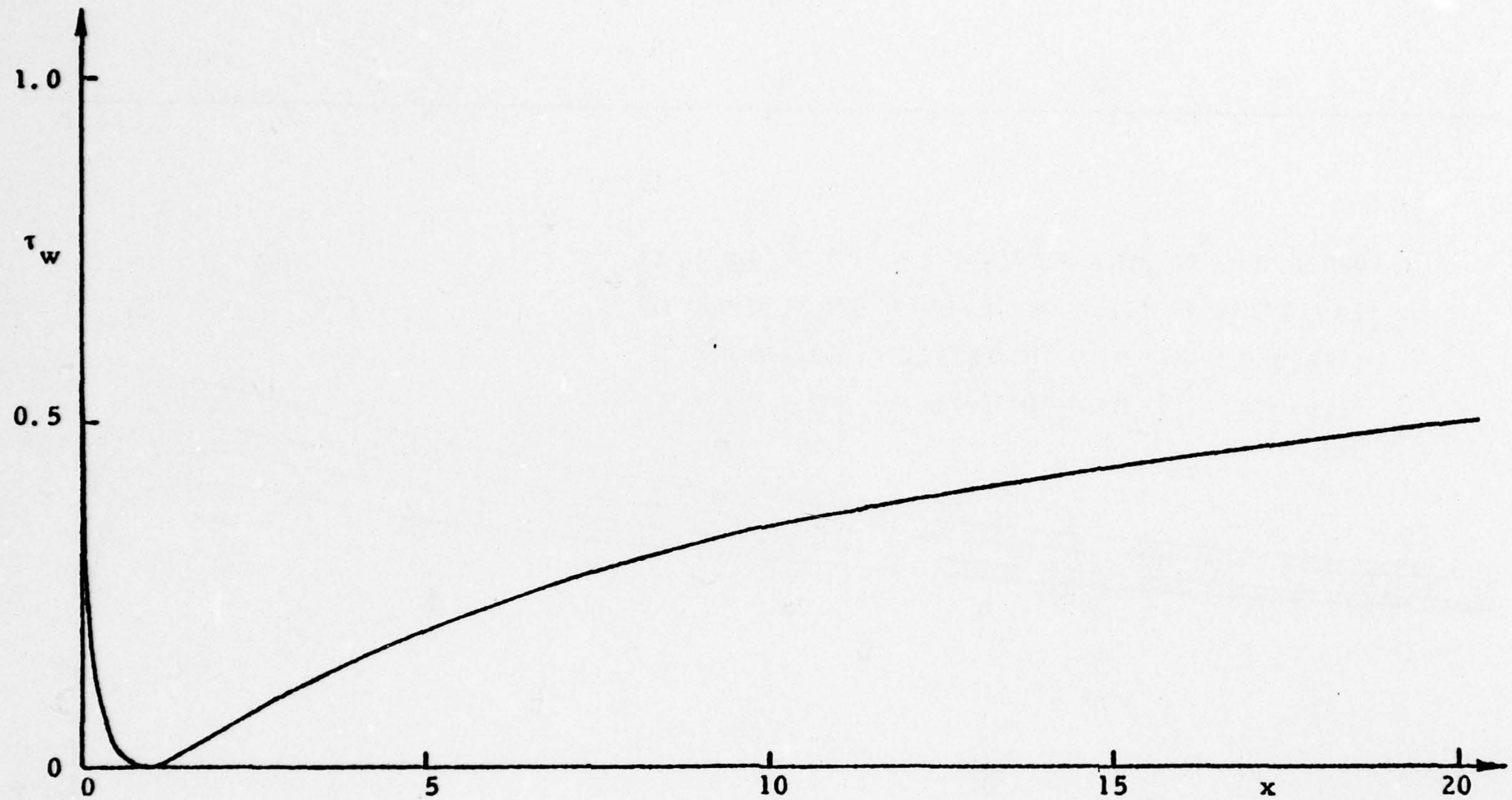

Figure 2

Wall shear stress at incipient separation for Re = 10^6 and ϵ = 0.2 (from Ref. 6)

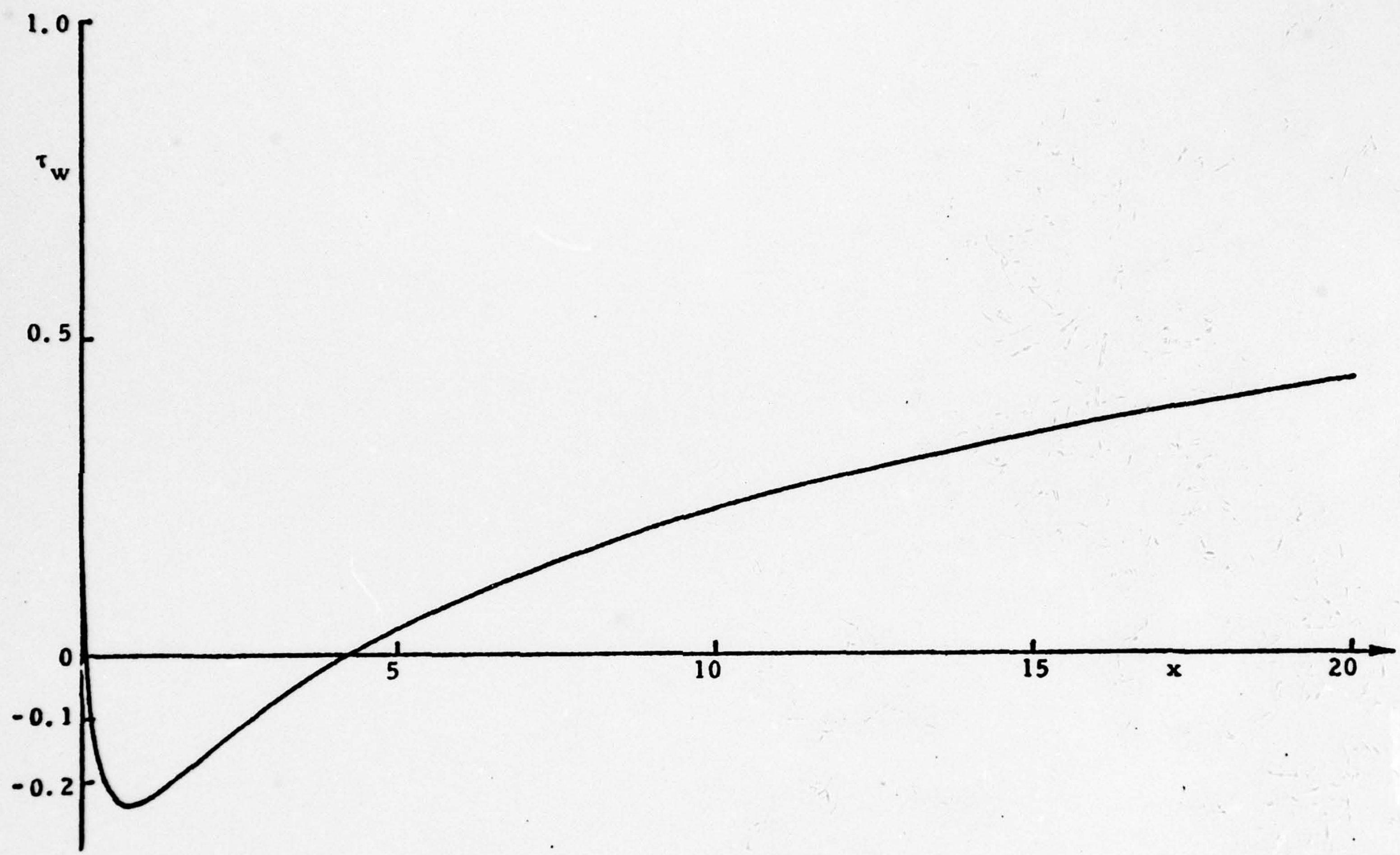

Figure 3

Wall shear stress for separated flow for $\epsilon = 0.25$, $Re = 10^6$ (from Ref. 6)